# MOSCOW STATE INSTITUTE OF INTERNATIONAL RELATIONS (MGIMO-UNIVERSITY), THE MINISTRY OF FOREIGN AFFAIRS OF RUSSIA

# B.A. in Government and International Affairs School of Government and International Affairs

# Department for Mathematics, Econometrics and IT

«Approved»

Dean of School of Government and International Affairs ——Y. I. Vaslavskiy

Рабочая программа дисциплины

**Quantitative Methods** 

(Наименование дисциплины)

Направление подготовки 41.03.05 Международные отношения

(шифр, название направления) Наименование образовательной программы

Международные отношения и управление

(название программы)

Квалификации (степень) выпускника Бакалавр Форма обучения

очная

(Очная, очно-заочная и др.)

The course program developed by Nikita V. Artamonov, Ph.D., 2015

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"Government and International Affairs")		ų o
Author		_ Nikita V. Artamonov, Ph.D
Director of the MGIMO library	feer-	Marina Reshetnikova
The program is approved by Department for	or Mathematics, eco	onometrics and IT on
Head of the Department		_Nikita V. Artamonov, Ph.D.

The course "Quantitative methods in Economics" is elaborated in accordance with MGIMO Educational Standard for the Bachelor's Program in International Affairs (program track

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# PART 1: INSTRUCTOR INFORMATION, COURSE DESCRIPTION AND TEACHING METHODS

### 1.1 General information

• Full course title: Quantitative Methods

• Type of course: Compulsory

• Level of course B.A.

Year of study: 1<sup>st</sup>

Number of ECTS credits allocated: 4

• Name of lecturer(s) and office hours:

Dr. Nikita V. Artamonov, Ph.D.

Associate Professor, Department for Econometrics and Mathematical Methods in Economics

Office hours: Friday 12PM-12:30PM, office 4028

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# 1.2 Course aims and learning outcomes

The purpose of the course is to provide a basic introduction to widely used quantitative, mathematical, quantitative and statistical methods for management, economics and business. The course is designed to introduce to quantitative principles and techniques for supporting decision-making, planning and public administration. We emphasize on learning to select the appropriate model or methods, applying the chosen technique and interpreting the obtained solution. We hit to give an intuition for determining situations where quantitative methods are appropriate to use and to prepare students for real application.

#### Learning outcomes:

By the end of this course students should be able to:

- know basic notions of linear algebra, calculus, optimization, probability and game theory.
- understand quantitative analysis of problem-solving and decision-making.
- identify and define the problem.
- select appropriate model or methods, applying the chosen technique and interpreting the obtained solution.
- use the computer technology, including spreadsheets.
- think critically about the uses and limitations of quantitative methods.

### 1.3 Course requirements and grading plan

### **Course requirements**

Students will be required to attend not less than 80% of classes and be prepared for class discussions. Conscientious reading of the assigned materials is compulsory. Students will also be required to hand on time required works (homework sets etc.). Late works will not be accepted. Students should be familiar with basic mathematical notions and computer skill (MS Word and MS Excel).

### Grading plan

• Class participation – 10%

Students are expected to attend lectures and seminars, participate in class discussions, asking clarifying questions and solving exercises; since the course is highly interactive, it is essential that students attend the seminars having read the materials for that day's class.

• In-class tests and midterm – 20%

Students will write three tests and one midterm during the semester. Dates of test classes are defined by the instructor and communicated to the students at the first class of the respective course.

- Homework assignments 20%
- Written final test 50% (in-class, 120 minutes).

# **PART 2. COURSE CONTENTS**

# 2.1Type of works

Types of work	Academic hours		
Total	72		
Total for classwork	32		
Lectures	16		
Seminars	16		
Homework	20		
Essay	0		
Preparation for lectures, seminars, written tests, home works	20		
Course assessment	Tests, activity at seminars, home works		

#### 2.2 Course outline

Section and topic	Full-time course				
	Academic hours taken (or credits)				
	Lectures	Seminars	Homework (incl. essay)	Total hours for topic	
1. Linear algebra	4	4	28	36	
2. Functions of one variable	4	4	28	36	
3. Functions of several variables.	2	2	14	18	
4.Unconstrained and constrained optimization	2	2	14	18	
5. Introduction to probability theory and statistical methods	2	2	14	18	
6. Introduction to decision theory and to game theory	2	2	14	18	
TOTAL	16	16	112	144	

### 2.3 Course contents

# Topic 1. Linear algebra: matrices, system of linear equation, linear space

Lecture 1.1Introduction. Matrices and linear spaces.

Summary: Introduction to the course, the course objective and links to economics and management. Matrices and vectors. Operation on matrices and vectors. Linear space.

Seminar 1.1Introduction.Matrices and linear spaces.

• Compulsoryreadings:

- o Thomas S. Shores, Applied Linear Algebra and Matrix Analysis. . Undergraduate Texts in Mathematics, Springer, 2007.
- Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.

http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

# • Further readings:

o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. - Springer Finance, 2009.

# • Internet resources:

- o http://imai.princeton.edu/index.html
- o http://ocw.mit.edu/index.htm
- o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
- o http://mat.gsia.cmu.edu/classes/QUANT/
- o <a href="http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/">http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/</a>
- o http://ocw.mit.edu/courses/mathematics/18-700-linear-algebra-fall-2005/

#### Lecture 1.2 Matrices.

Summary: Determinants of square matrices. Inverse matrix.

Seminar 1.2 Matrices.

# • Compulsory readings:

- o Thomas S. Shores, Applied Linear Algebra and Matrix Analysis. . Undergraduate Texts in Mathematics, Springer, 2007.
- Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.

http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

### • Further readings:

 Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. -Springer Finance, 2009.

#### • Internet resources:

- o <a href="http://imai.princeton.edu/index.html">http://imai.princeton.edu/index.html</a>
- o http://ocw.mit.edu/index.htm
- o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
- o http://mat.gsia.cmu.edu/classes/QUANT/
- o http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/
- o http://ocw.mit.edu/courses/mathematics/18-700-linear-algebra-fall-2005/

### Lecture 1.3 Linear equations.

Summary: Systems of linear equations. Matrix equations

Seminar 1.3 Linear equations.

# Compulsory readings:

- o Thomas S. Shores, Applied Linear Algebra and Matrix Analysis. . Undergraduate Texts in Mathematics, Springer, 2007.
- Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.

http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

#### Further readings:

o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. - Springer Finance, 2009.

#### • Internet resources:

- o http://imai.princeton.edu/index.html
- o <a href="http://ocw.mit.edu/index.htm">http://ocw.mit.edu/index.htm</a>
- o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf

- o http://mat.gsia.cmu.edu/classes/QUANT/
- o http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/
- o http://ocw.mit.edu/courses/mathematics/18-700-linear-algebra-fall-2005/

# Lecture 1.4 Linear equations.

Summary: Systems of linear equations, matrix equations (continuation). Positive definite matrices.

Seminar 1.4 Linear equations.

- Compulsory readings:
  - o Thomas S. Shores, Applied Linear Algebra and Matrix Analysis. . Undergraduate Texts in Mathematics, Springer, 2007.
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850
- Further readings:
  - o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
- Internet resources:
  - o http://imai.princeton.edu/index.html
  - o http://ocw.mit.edu/index.htm
  - o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
  - o http://mat.gsia.cmu.edu/classes/QUANT/
  - o http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/
  - o http://ocw.mit.edu/courses/mathematics/18-700-linear-algebra-fall-2005/

# **Topic 2.** Functions of one variable

Lecture 2.1 Derivatives.

Summary: Derivative of one variable functions. Monotonic functions.

Seminar 2.1 Derivatives.

- Compulsory readings:
  - o Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.

http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

- Further readings:
  - o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
  - o Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.
- Internet resources:
  - o http://imai.princeton.edu/index.html
  - o http://ocw.mit.edu/index.htm
  - o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
  - o http://mat.gsia.cmu.edu/classes/OUANT/
  - o <a href="http://ocw.mit.edu/courses/mathematics/18-01-single-variable-calculus-fall-2006/index.htm">http://ocw.mit.edu/courses/mathematics/18-01-single-variable-calculus-fall-2006/index.htm</a>
  - o http://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/
  - o http://ocw.mit.edu/courses/mathematics/18-014-calculus-with-theory-fall-2010/
  - <a href="http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/</a>

o http://ocw.mit.edu/courses/mathematics/18-024-calculus-with-theory-ii-spring-2003/

Lecture 2.2 Convexity and local optima.

Summary: Derivative of one variable functions (continuation). Convex functions. Local minimum and maximum of one variable functions.

Seminar 2.2 Convexity and local optima.

- Compulsory readings:
  - o Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.

http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

- Further readings:
  - o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
  - o Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.
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  - o <a href="http://imai.princeton.edu/index.html">http://imai.princeton.edu/index.html</a>
  - o http://ocw.mit.edu/index.htm
  - o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
  - o http://mat.gsia.cmu.edu/classes/QUANT/
  - o <a href="http://ocw.mit.edu/courses/mathematics/18-01-single-variable-calculus-fall-2006/index.htm">http://ocw.mit.edu/courses/mathematics/18-01-single-variable-calculus-fall-2006/index.htm</a>
  - o http://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/
  - o http://ocw.mit.edu/courses/mathematics/18-014-calculus-with-theory-fall-2010/
  - o <a href="http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/</a>
  - o http://ocw.mit.edu/courses/mathematics/18-024-calculus-with-theory-ii-spring-2003/

# Lecture 2.3 Local and global optima.

Summary: Local minimum and maximum of one variable functions (continuation). Global optima.

Seminar 2.3 Local and global optima.

- Compulsory readings:
  - o Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.

http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

- Further readings:
  - o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
  - o Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.
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  - o http://ocw.mit.edu/index.htm
  - o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
  - o http://mat.gsia.cmu.edu/classes/QUANT/
  - o <a href="http://ocw.mit.edu/courses/mathematics/18-01-single-variable-calculus-fall-2006/index.htm">http://ocw.mit.edu/courses/mathematics/18-01-single-variable-calculus-fall-2006/index.htm</a>

- o http://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/
- o <a href="http://ocw.mit.edu/courses/mathematics/18-014-calculus-with-theory-fall-2010/">http://ocw.mit.edu/courses/mathematics/18-014-calculus-with-theory-fall-2010/</a>
- o <a href="http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/</a>
- o <a href="http://ocw.mit.edu/courses/mathematics/18-024-calculus-with-theory-ii-spring-2003/">http://ocw.mit.edu/courses/mathematics/18-024-calculus-with-theory-ii-spring-2003/</a>

### **Topic 3.** Functions of several variables

Lecture 3.1 Partial derivatives.

*Summary*: Partial derivative of several variable functions. Gradient and Hessian. Convexity. Seminar 3.1 Partial derivatives.

- Compulsory readings:
  - o Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850
- Further readings:
  - o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
  - Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.
- Internet resources:
  - o http://imai.princeton.edu/index.html
  - o http://ocw.mit.edu/index.htm
  - o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
  - o http://mat.gsia.cmu.edu/classes/OUANT/
  - <a href="http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/</a>
  - o http://ocw.mit.edu/courses/mathematics/18-024-calculus-with-theory-ii-spring-2003/
  - o <a href="http://ocw.mit.edu/courses/mathematics/18-02-multivariable-calculus-fall-2007/index.htm">http://ocw.mit.edu/courses/mathematics/18-02-multivariable-calculus-fall-2007/index.htm</a>
  - o <a href="http://ocw.mit.edu/courses/mathematics/18-02sc-multivariable-calculus-fall-2010/index.htm">http://ocw.mit.edu/courses/mathematics/18-02sc-multivariable-calculus-fall-2010/index.htm</a>
  - o http://ocw.mit.edu/courses/mathematics/18-022-calculus-of-several-variables-fall-2010/

Lecture 3.2 Local optima for functions of several variables.

Summary: Partial derivative of several variable functions (continuation). Local optima.

Seminar 3.2 Local optima for functions of several variables.

- Compulsory readings:
  - o Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850
- Further readings:
  - Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. -Springer Finance, 2009.
  - Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.
- Internet resources:

- o http://imai.princeton.edu/index.html
- o http://ocw.mit.edu/index.htm
- o <a href="http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf">http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf</a>
- o http://mat.gsia.cmu.edu/classes/QUANT/
- o <a href="http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/</a>
- o http://ocw.mit.edu/courses/mathematics/18-024-calculus-with-theory-ii-spring-2003/
- o <a href="http://ocw.mit.edu/courses/mathematics/18-02-multivariable-calculus-fall-2007/index.htm">http://ocw.mit.edu/courses/mathematics/18-02-multivariable-calculus-fall-2007/index.htm</a>
- o <a href="http://ocw.mit.edu/courses/mathematics/18-02sc-multivariable-calculus-fall-2010/index.htm">http://ocw.mit.edu/courses/mathematics/18-02sc-multivariable-calculus-fall-2010/index.htm</a>
- o http://ocw.mit.edu/courses/mathematics/18-022-calculus-of-several-variables-fall-2010/

# Topic 4. Unconstrained and constrained optimization

Lecture 4.1 Lagrange multipliers methods.

Summary: Unconstrained and constrained optimization. Equality and inequality constraints. Lagrange multipliers methods.

Seminar 4.1 Lagrange multipliers methods.

- o Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.
- Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.
  - http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850
- Further readings:
  - o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
  - o Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.
- Internet resources:
  - o http://imai.princeton.edu/index.html
  - o <a href="http://ocw.mit.edu/index.htm">http://ocw.mit.edu/index.htm</a>
  - o <a href="http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf">http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf</a>
  - o <a href="http://mat.gsia.cmu.edu/classes/QUANT/">http://mat.gsia.cmu.edu/classes/QUANT/</a>
  - o http://homepages.stmartin.edu/fac staff/dstout/MBA605/lecture notes.htm
  - <a href="http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/</a>
  - o http://ocw.mit.edu/courses/mathematics/18-024-calculus-with-theory-ii-spring-2003/

Lecture 4.2 Convex programming.

Summary: Lagrange multipliers methods (continuation). Convex programming and global optima.

Seminar 4.2 Convex programming.

- Compulsory readings:
  - o Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.
    - http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850
- Further readings:

- o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
- o Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.

#### • Internet resources:

- o http://imai.princeton.edu/index.html
- o http://ocw.mit.edu/index.htm
- o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
- o http://mat.gsia.cmu.edu/classes/QUANT/
- o http://homepages.stmartin.edu/fac staff/dstout/MBA605/lecture notes.htm
- o <a href="http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-013a-calculus-with-applications-spring-2005/</a>
- o http://ocw.mit.edu/courses/mathematics/18-024-calculus-with-theory-ii-spring-2003/

# Lecture 4.3 Linear programming and applications.

*Summary*: Linear programming, modeling with linear programming. Solving with spreadsheets. Examples.

Seminar 4.3 Linear programming and applications.

# • Compulsory readings:

- o Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.
- Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

# • Further readings:

- o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
- o Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.

### • Internet resources:

- o http://imai.princeton.edu/index.html
- o http://ocw.mit.edu/index.htm
- o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
- o http://mat.gsia.cmu.edu/classes/QUANT/
- o http://homepages.stmartin.edu/fac staff/dstout/MBA605/lecture notes.htm
- $\verb| \underline{http://ocw.mit.edu/courses/mathematics/18-310c-principles-of-applied-mathematics-fall-2007/} \\$
- o <a href="http://ocw.mit.edu/courses/mathematics/18-086-mathematical-methods-for-engineers-ii-spring-2006/index.htm">http://ocw.mit.edu/courses/mathematics/18-086-mathematical-methods-for-engineers-ii-spring-2006/index.htm</a>
- o <a href="http://ocw.mit.edu/courses/mathematics/18-310c-principles-of-applied-mathematics-fall-2007/">http://ocw.mit.edu/courses/mathematics/18-310c-principles-of-applied-mathematics-fall-2007/</a>
- http://ocw.mit.edu/courses/sloan-school-of-management/15-093j-optimization-methods-fall-2009/lecture-notes/

# Topic 5. Introduction to probability theory and statistical methods

Lecture 5.1 Introduction to probability.

Summary: Introduction to probability theory. Outcomes and probability space. Combinatoric formulae. Events and operations on events.

Seminar 5.1 Introduction to probability.

• Compulsory readings:

- Anirban DasGupta, Fundamentals of Probability: A First Course. Springer Texts in Statistics, 2010.
- Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

# Further readings:

- o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
- o Mario Lefebvre, Basic Probability theory with Applications. Springer Undergarduate Texts in Mathematics and Technology, 2009.
- o Rabi Bhattacharya, Edward C. Waymire, A Basic Course in Probability Theory. Universitext, Springer, 2007.

# • Internet resources:

- o http://imai.princeton.edu/index.html
- o http://ocw.mit.edu/index.htm
- o http://mat.gsia.cmu.edu/classes/QUANT/
- http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-436jfundamentals-of-probability-fall-2008/
- o <a href="http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-spring-2006/">http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-spring-2006/</a>
- http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2009/
- o <a href="http://ocw.mit.edu/courses/civil-and-environmental-engineering/1-151-probability-and-statistics-in-engineering-spring-2005/index.htm">http://ocw.mit.edu/courses/civil-and-environmental-engineering/1-151-probability-and-statistics-in-engineering-spring-2005/index.htm</a>
- o http://ocw.mit.edu/courses/mathematics/18-175-theory-of-probability-fall-2008/
- http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statisticsspring-2005/
- o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-075-applied-statistics-spring-2003/index.htm">http://ocw.mit.edu/courses/sloan-school-of-management/15-075-applied-statistics-spring-2003/index.htm</a>

#### Lecture 5.2 Events and probability formulas.

Summary: Events and operations on events (continuation). Probability of sum and of product of events. Bias formula. Independence, conditional probability.

Seminar 5.2 Events and probability formulas.

# Compulsory readings:

- Anirban DasGupta, Fundamentals of Probability: A First Course. Springer Texts in Statistics, 2010.
- Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.
- http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

#### • Further readings:

- o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
- o Mario Lefebvre, Basic Probability theory with Applications. Springer Undergarduate Texts in Mathematics and Technology, 2009.
- O Rabi Bhattacharya, Edward C. Waymire, A Basic Course in Probability Theory. Universitext, Springer, 2007.

### • Internet resources:

- o http://imai.princeton.edu/index.html
- o http://ocw.mit.edu/index.htm
- o http://mat.gsia.cmu.edu/classes/QUANT/

- o <a href="http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-436j-fundamentals-of-probability-fall-2008/">http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-436j-fundamentals-of-probability-fall-2008/</a>
- o <a href="http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-spring-2006/">http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-spring-2006/</a>
- o <a href="http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2009/">http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2009/</a>
- o <a href="http://ocw.mit.edu/courses/civil-and-environmental-engineering/1-151-probability-and-statistics-in-engineering-spring-2005/index.htm">http://ocw.mit.edu/courses/civil-and-environmental-engineering/1-151-probability-and-statistics-in-engineering-spring-2005/index.htm</a>
- o http://ocw.mit.edu/courses/mathematics/18-175-theory-of-probability-fall-2008/
- o <a href="http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2005/</a>
- o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-075-applied-statistics-spring-2003/index.htm">http://ocw.mit.edu/courses/sloan-school-of-management/15-075-applied-statistics-spring-2003/index.htm</a>

#### Lecture 5.3 Random variables.

Summary: Random variable, distribution of random variable. Expectation and variance of random variable.

Seminar 5.3 Random variables.

# Compulsory readings:

- o Anirban DasGupta, Fundamentals of Probability: A First Course. Springer Texts in Statistics, 2010.
- Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

# • Further readings:

- o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
- o Mario Lefebvre, Basic Probability theory with Applications. Springer Undergarduate Texts in Mathematics and Technology, 2009.
- o Rabi Bhattacharya, Edward C. Waymire, A Basic Course in Probability Theory. Universitext, Springer, 2007.

#### Internet resources:

- o http://imai.princeton.edu/index.html
- o http://ocw.mit.edu/index.htm
- o http://mat.gsia.cmu.edu/classes/QUANT/
- http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-436jfundamentals-of-probability-fall-2008/
- o <a href="http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-spring-2006/">http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-spring-2006/</a>
- o <a href="http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2009/">http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2009/</a>
- http://ocw.mit.edu/courses/civil-and-environmental-engineering/1-151-probability-and-statistics-in-engineering-spring-2005/index.htm
- o http://ocw.mit.edu/courses/mathematics/18-175-theory-of-probability-fall-2008/
- o <a href="http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2005/</a>
- http://ocw.mit.edu/courses/sloan-school-of-management/15-075-applied-statisticsspring-2003/index.htm

#### Lecture 5.4 Bivariate random variable.

Summary: Bivariate random variable, joint and marginal distributions. Function of random variables and their expectation. Conditional distribution.

Seminar 5.4 Bivariate random variable.

- Compulsory readings:
  - o Anirban DasGupta, Fundamentals of Probability: A First Course. Springer Texts in Statistics, 2010.
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

# Further readings:

- o Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
- o Mario Lefebvre, Basic Probability theory with Applications. Springer Undergarduate Texts in Mathematics and Technology, 2009.
- o Rabi Bhattacharya, Edward C. Waymire, A Basic Course in Probability Theory. Universitext, Springer, 2007.

#### • Internet resources:

- o http://imai.princeton.edu/index.html
- o http://ocw.mit.edu/index.htm
- o http://mat.gsia.cmu.edu/classes/QUANT/
- o <a href="http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-436j-fundamentals-of-probability-fall-2008/">http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-436j-fundamentals-of-probability-fall-2008/</a>
- o <a href="http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-spring-2006/">http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-spring-2006/</a>
- o <a href="http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2009/">http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2009/</a>
- o <a href="http://ocw.mit.edu/courses/civil-and-environmental-engineering/1-151-probability-and-statistics-in-engineering-spring-2005/index.htm">http://ocw.mit.edu/courses/civil-and-environmental-engineering/1-151-probability-and-statistics-in-engineering-spring-2005/index.htm</a>
- http://ocw.mit.edu/courses/mathematics/18-175-theory-of-probability-fall-2008/
- <a href="http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2005/">http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2005/</a>
- <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-075-applied-statistics-spring-2003/index.htm">http://ocw.mit.edu/courses/sloan-school-of-management/15-075-applied-statistics-spring-2003/index.htm</a>

# Topic 6. Introduction to decision theory and to game theory

Lecture 6.1 Basic notions of game theory.

Summary: Introduction to decision theory and game theory. Normal form of the game. Pure strategies, Nash equilibrium, Pareto equilibrium.

Seminar 6.1 Basic notions of game theory.

- Compulsory readings:
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.
    - http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850
- Further readings:
  - o Marisa Faggini, Concetto Paolo Vinci, Decision Theory and Choices: a Complexity Approach. New Economic Window, Springer, 2010.
- Internet resources:
  - o http://imai.princeton.edu/index.html
  - o <a href="http://ocw.mit.edu/index.htm">http://ocw.mit.edu/index.htm</a>
  - o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
  - o http://mat.gsia.cmu.edu/classes/QUANT/
  - o http://homepages.stmartin.edu/fac staff/dstout/MBA605/lecture notes.htm

- o http://ocw.mit.edu/courses/economics/14-147-topics-in-game-theory-fall-2009/
- o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-040-game-theory-for-managers-spring-2004/">http://ocw.mit.edu/courses/sloan-school-of-management/15-040-game-theory-for-managers-spring-2004/</a>
- o <a href="http://ocw.mit.edu/courses/economics/14-147-topics-in-game-theory-spring-2005/">http://ocw.mit.edu/courses/economics/14-147-topics-in-game-theory-spring-2005/</a>
- o <a href="http://ocw.mit.edu/courses/economics/14-12-economic-applications-of-game-theory-fall-2005/">http://ocw.mit.edu/courses/economics/14-12-economic-applications-of-game-theory-fall-2005/</a>
- o http://ocw.mit.edu/courses/economics/14-126-game-theory-spring-2010/
- o <a href="http://ocw.mit.edu/courses/political-science/17-881-game-theory-and-political-theory-fall-2004/">http://ocw.mit.edu/courses/political-science/17-881-game-theory-and-political-theory-fall-2004/</a>
- o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-060-data-models-and-decisions-fall-2007/index.htm">http://ocw.mit.edu/courses/sloan-school-of-management/15-060-data-models-and-decisions-fall-2007/index.htm</a>
- o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-067-competitive-decision-making-and-negotiation-spring-2003/">http://ocw.mit.edu/courses/sloan-school-of-management/15-067-competitive-decision-making-and-negotiation-spring-2003/</a>
- o <a href="http://ocw.mit.edu/courses/linguistics-and-philosophy/24-222-decisions-games-and-rational-choice-spring-2008/">http://ocw.mit.edu/courses/linguistics-and-philosophy/24-222-decisions-games-and-rational-choice-spring-2008/</a>

# Lecture 6.2 Nash equilibrium.

Summary: Introduction to decision theory and game theory (continuation). Mixed strategies, Nash equilibrium in mixed strategies.

Seminar 6.2 Nash equilibrium.

- Compulsory readings:
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850
- Further readings:
  - o Marisa Faggini, Concetto Paolo Vinci, Decision Theory and Choices: a Complexity Approach. New Economic Window, Springer, 2010.
- Internet resources:
  - o http://imai.princeton.edu/index.html
  - o http://ocw.mit.edu/index.htm
  - o <a href="http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf">http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf</a>
  - o http://mat.gsia.cmu.edu/classes/QUANT/
  - o http://homepages.stmartin.edu/fac staff/dstout/MBA605/lecture notes.htm
  - o http://ocw.mit.edu/courses/economics/14-147-topics-in-game-theory-fall-2009/
  - o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-040-game-theory-for-managers-spring-2004/">http://ocw.mit.edu/courses/sloan-school-of-management/15-040-game-theory-for-managers-spring-2004/</a>
  - o http://ocw.mit.edu/courses/economics/14-147-topics-in-game-theory-spring-2005/
  - o <a href="http://ocw.mit.edu/courses/economics/14-12-economic-applications-of-game-theory-fall-2005/">http://ocw.mit.edu/courses/economics/14-12-economic-applications-of-game-theory-fall-2005/</a>
  - o http://ocw.mit.edu/courses/economics/14-126-game-theory-spring-2010/
  - o <a href="http://ocw.mit.edu/courses/political-science/17-881-game-theory-and-political-theory-fall-2004/">http://ocw.mit.edu/courses/political-science/17-881-game-theory-and-political-theory-fall-2004/</a>
  - o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-060-data-models-and-decisions-fall-2007/index.htm">http://ocw.mit.edu/courses/sloan-school-of-management/15-060-data-models-and-decisions-fall-2007/index.htm</a>
  - o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-067-competitive-decision-making-and-negotiation-spring-2003/">http://ocw.mit.edu/courses/sloan-school-of-management/15-067-competitive-decision-making-and-negotiation-spring-2003/</a>
  - o <a href="http://ocw.mit.edu/courses/linguistics-and-philosophy/24-222-decisions-games-and-rational-choice-spring-2008/">http://ocw.mit.edu/courses/linguistics-and-philosophy/24-222-decisions-games-and-rational-choice-spring-2008/</a>

# Lecture 6.2 Nash equilibrium.

Summary: Introduction to decision theory and game theory (continuation). Mixed strategies, Nash equilibrium in mixed strategies.

Seminar 6.2 Nash equilibrium.

- Compulsory readings:
  - Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp.

http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850

- Further readings:
  - o Marisa Faggini, Concetto Paolo Vinci, Decision Theory and Choices: a Complexity Approach. New Economic Window, Springer, 2010.
- Internet resources:
  - o http://imai.princeton.edu/index.html
  - o http://ocw.mit.edu/index.htm
  - o http://www.cse.iitd.ernet.in/~naveen/courses/optimization/trick.pdf
  - o http://mat.gsia.cmu.edu/classes/QUANT/
  - o http://homepages.stmartin.edu/fac staff/dstout/MBA605/lecture notes.htm
  - o http://ocw.mit.edu/courses/economics/14-147-topics-in-game-theory-fall-2009/
  - o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-040-game-theory-for-managers-spring-2004/">http://ocw.mit.edu/courses/sloan-school-of-management/15-040-game-theory-for-managers-spring-2004/</a>
  - o http://ocw.mit.edu/courses/economics/14-147-topics-in-game-theory-spring-2005/
  - o <a href="http://ocw.mit.edu/courses/economics/14-12-economic-applications-of-game-theory-fall-2005/">http://ocw.mit.edu/courses/economics/14-12-economic-applications-of-game-theory-fall-2005/</a>
  - o http://ocw.mit.edu/courses/economics/14-126-game-theory-spring-2010/
  - o <a href="http://ocw.mit.edu/courses/political-science/17-881-game-theory-and-political-theory-fall-2004/">http://ocw.mit.edu/courses/political-science/17-881-game-theory-and-political-theory-fall-2004/</a>
  - o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-060-data-models-and-decisions-fall-2007/index.htm">http://ocw.mit.edu/courses/sloan-school-of-management/15-060-data-models-and-decisions-fall-2007/index.htm</a>
  - o <a href="http://ocw.mit.edu/courses/sloan-school-of-management/15-067-competitive-decision-making-and-negotiation-spring-2003/">http://ocw.mit.edu/courses/sloan-school-of-management/15-067-competitive-decision-making-and-negotiation-spring-2003/</a>
  - <a href="http://ocw.mit.edu/courses/linguistics-and-philosophy/24-222-decisions-games-and-rational-choice-spring-2008/">http://ocw.mit.edu/courses/linguistics-and-philosophy/24-222-decisions-games-and-rational-choice-spring-2008/</a>

#### 2.4 Essay topics/Questions for testing

- 1. Matrices and operations on matrices, determinant, inverse matrix.
- 2. Linear spaces, subspaces.
- 3. System of linear equations.
- 4. Derivatives for functions of one variable.
- 5. Monotonic functions.
- 6. Convex and concave functions of one variable.
- 7. Local optima for functions of one variable.
- 8. Global optima for functions of one variable.
- 9. Partial derivatives for functions of several variables.
- 10. Convex functions of several variables
- 11. Local optima for functions of several variables.
- 12. Lagrange multipliers methods for equality constraint optimization.
- 13. Convex programming.
- 14. Linear programming.
- 15. Combinatorial formulae.
- 16. Events in theory of probability. Operations on events.
- 17. Conditional probability and independence.
- 18. Probability of sum and product of events.
- 19. Bias formula.
- 20. Random variable, distribution of random variable.

- 21. Expectation and variance of random variable.
- 22. Bivariante random variable. Joint and marginal distribution.
- 23. Function of random variables and their expectation.
- 24. Conditional distribution.
- 25. Normal form of game. Pure strategies.
- 26. Pure-strategy Nash equilibrium.
- 27. Mixed strategies.
- 28. Mixed Nash equilibrium.

### 2.5 Test and exam timing

- Fall semester tests last week of December; exams January 9-25
- Spring semester tests last week of May; exams June 1-25

# 2.6 Consolidated reading list (in alphabetic order)

Compulsory readings

- 1. Anirban DasGupta, Fundamentals of Probability: A First Course. Springer Texts in Statistics, 2010.
- 2. Giaquinta M., Modica G. Mathematical Analysis: An Introduction to Functions of Several Variables. Springer, 2009.

3.

- 4. Paolo Brandimarte, Quantitative Methods: An Introduction for Business Management, Wiley, 2011, 912 pp. <a href="http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850">http://site.ebrary.com/lib/mgimo/docDetail.action?docID=10469850</a>
- 5. Thomas S. Shores, Applied Linear Algebra and Matrix Analysis. . Undergraduate Texts in Mathematics, Springer, 2007.

Further readings

- 1. Jeanblanc M., Yor M., Chesney M. Mathematical Methods for Financial Markets. Springer Finance, 2009.
- 2. Mario Lefebvre, Basic Probability theory with Applications. Springer Undergarduate Texts in Mathematics and Technology, 2009.
- 3. Marisa Faggini, Concetto Paolo Vinci, Decision Theory and Choices: a Complexity Approach.

   New Economic Window, Springer, 2010.
- 4. Omar Hijab, Introduction to Calculus and Classical Analysis. Undergraduate Texts in Mathematics, Springer, 2007.

5.

6. Rabi Bhattacharya, Edward C. Waymire, A Basic Course in Probability Theory. – Universitext, Springer, 2007.

#### **PART 3. FINAL REMARKS**

- Plagiarism is considered as a severe violation and as an indication of incompetence in the
  course. Plagiarism is understood as making of one's text using compilation method for other
  people's publications, even connected with own phrases and sentences. Collective performance
  of individual tasks is also unacceptable. Proved plagiarism leads to an F-mark regardless of the
  fulfillment of all other requirements.
- Assignments are to be handed in on the due date. Late submissions will not be accepted.
- Students are asked to keep a copy of all work submitted for evaluation.
- All work (including home assignments, tests, final test, etc.) must be the result of students' own efforts and labor.